Advances in Transmit Beamforming Techniques for MIMO Radar

Chippa Hritika Ravindra, Chaudhari Sarthak Sanjay, Deshpande Nishant Sachin, Yerram Ravinder

SCTR's Pune Institute of Computer Technology, (E&TC), Pune, Maharashtra, India

Corresponding author: Chaudhari Sarthak Sanjay, Email: sarthakchaudhari401@gmail.com

MIMO radar uses multiple antennas for transmitting and receiving signals, enabling advanced signal processing to improve target detection, localization, and tracking. By employing beamforming techniques, MIMO radar can focus signals on specific directions, enhancing accuracy and robustness. Various adaptive beamforming algorithms are available to optimize the beam pattern, achieving maximum gain in the desired direction while minimizing interference. Comparing different techniques and steering directions allows us to evaluate beamforming effectiveness, while analysing signal-to-noise ratio (SNR) and target detection performance helps determine the optimal configuration for specific applications.

Keywords: MIMO Radar, Beam Forming, Transmit Beam Formation, convex optimization, Sequential quadratic programming, Signal cross correlation, gradient descent.

2025. In Kusum Kumari Bharti & Noor Firdoos Jahan (eds.), *Next-Gen Data Analytics and Intelligent Automation*, 1–10. Computing & Intelligent Systems, SCRS, India. DOI: https://doi.org/10.56155/978-81-975670-6-3-1

1 Introduction

Beamforming is a signal processing technique that directs signal transmission or reception in a specific direction using an array of antennas, significantly improving signal strength by focusing energy on a target and reducing interference from unwanted sources. This enhances the signal-to-noise ratio (SNR) and overall system efficiency, making it applicable across various domains like telecommunications, radar, and sonar. In MIMO radar, beamforming optimizes the power and direction of signals transmitted across multiple antennas, improving detection accuracy and target tracking. Unlike phased arrays, which transmit scaled versions of the same waveform, MIMO systems allow each antenna to transmit diverse waveforms, offering improved spatial resolution [5]. Recent research on transmit beamforming for uniform linear arrays demonstrates its effectiveness across different scenarios [7]. Additionally, adaptive beamforming techniques like Gradient Descent Algorithm, Signal Cross Correlation [4], Sequential Quadratic Programming, and Semidefinite Relaxation [10] enable dynamic control of radiation patterns, optimizing target detection. Our work also extends to beamforming for planar arrays, which offer better spatial coverage and improved performance in multi-target environments. Beamforming continues to evolve with advancements in artificial intelligence and machine learning, enabling more efficient and adaptive signal processing techniques for complex and dynamic environments. Beamforming techniques are increasingly being integrated with machine learning algorithms to further enhance adaptability and performance in dynamic environments. These advancements have opened up new possibilities for applications in autonomous vehicles, IoT networks, and advanced healthcare systems.

2 Proposed Research Work

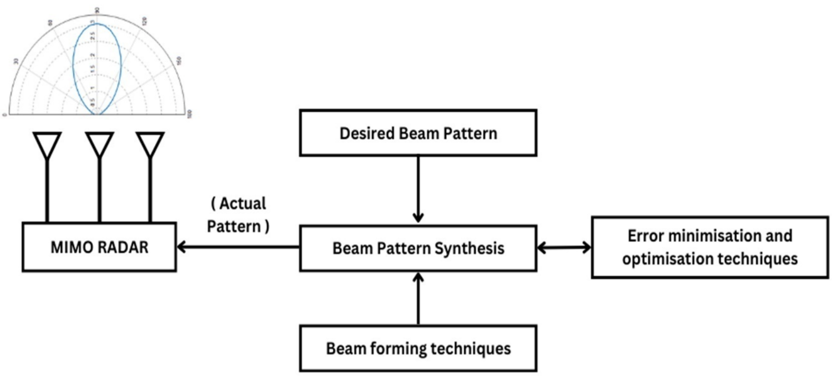

Figure 1. Block Diagram

Figure 1 presents a block diagram detailing the signal processing workflow for beam pattern optimization in a MIMO radar system. The process begins with the MIMO radar generating the actual beam pattern, which is subsequently input into the beam pattern synthesis block. This block, informed by advanced beamforming techniques, manipulates the beam pattern to approximate a desired configuration. The desired beam pattern is defined based on system objectives, and error minimization and optimization techniques are employed to reduce deviations between the actual and desired patterns. The closed-loop architecture ensures iterative refinement, enhancing the system's ability to suppress interference and focus energy on the intended direction, ultimately optimizing radar performance. This framework enables the system to dynamically adapt to changing operational environments and target scenarios.

3 Beamforming Algorithms

3.1 Gradient Descent Algorithm

Gradient descent is a key optimization algorithm used to adjust beamforming weights to minimize errors, such as mean square error (MSE) [30]. The steering vector, representing phase shifts and amplitudes for different directions in the radar array, models how signals combine spatially. In transmit beamforming, gradient descent iteratively adjusts the steering vector's weights to direct energy toward desired directions while suppressing interference from others, optimizing performance.

ALGORITHM

1. This step involves initialization of the steering vector $A(\theta)$, this steering vector is based on the array geometry and the target direction denoted by angle θ . Additionally set the convergence point and the rate of change
2. Here we define a cost function which is denoted by *f(R)*. This cost function represents the error between the desired and the obtained beam pattern.
3. In this step we calculate the gradient of the cost function $\nabla f(R(\theta))$, this gradient indicates the manner in which the matrix must vary in order to reduce the error in the beam pattern.
4. Update the matrix by moving in the opposite direction of the gradient
$$R_{\text{new}} = R_{\text{old}} - \nabla f(R(\theta)) \quad (1)$$
Repeat steps 2-4 until the change in the cost function *f(R)* is smaller than the convergence threshold ϵ, meaning that the beam pattern is optimised, and the angle has converged to the optimal value.
5. Once the process converges, the final angle $_{\text{opt}}$ directs the transmitted beam optimally towards the desired target direction, improving detection or performance.

3.2 Signal Cross Correlation

Signal cross-correlation measures the similarity between two signals and, in MIMO radar, enhances capabilities by manipulating the signal cross-correlation matrix R to control beam patterns [8]. Unlike traditional methods that reduce transmit power, this approach preserves full power while adjusting the directionality of the beam. It allows flexibility between phased-array directionality and MIMO omnidirectionality. The radiated signal is expressed as,

$$f(\boldsymbol{L}) = \int_{\theta} \left(P_d^{1/2}(\theta) - |\boldsymbol{a}^{\boldsymbol{H}}\boldsymbol{L}|\right)^2 \cos(\theta)\, d\theta \quad (2)$$

In [4,8], the authors show how to compute an ideal signal correlation matrix R using a cost function to optimize the beam pattern. The signal model is described by,

$$y(t, r, \theta, \varphi) = \frac{1}{\sqrt{4\pi r}} \sum_{i=1}^{N} s_i\left(t - \frac{r}{c}\right) e^{j\frac{2\pi z_i}{\lambda}\sin\theta} \quad (3)$$

where the cross-correlation matrix R must be nonnegative definite. The goal is to optimize R for the best beamforming performance.

ALGORITHM

Let N be the number of transmitters, and the signals transmitted by them be si(t) and define the signal cross correlation matrix, R which is a positive semi-definite matrix.

1. The spatial power density is expressed as,

$$P(\theta)\ \frac{1}{4\pi}a^H(\theta)R\,a(\theta) \quad (4)$$

here, $a(\theta)$ represents the steering vector which is a function of angle θ

2. Set the cost function, $J(R)$ to minimize the difference between desired and actual beampatterns.

$$J(\boldsymbol{L}) = \int_\theta \left(P_d^{1/2}(\theta) - |\boldsymbol{a^H L}|\right)^2 \cos(\theta)\ d\theta \ (5)$$

3. Using the gradient descent method, update R as,

$$R_{k+1} = R_k - \eta\Delta RJ(R_k) \quad (6)$$

4. Upon convergence we get the following as the optimized beampattern:

$$P(\theta) = \frac{1}{4\pi}a^H(\theta)R_{opt}\,a(\theta) \quad (7)$$

3.3 Sequential Quadratic Programming

Sequential Quadratic Programming (SQP) is an iterative optimization method used in MIMO radar systems for tasks like waveform design, beamforming, and resource allocation. It works by solving a series of quadratic programming (QP) subproblems that approximate the original nonlinear problem. In adaptive beamforming, SQP minimizes an objective function while satisfying system constraints, iteratively refining the beamformer's steering vector to correct mismatches between presumed and actual steering vectors. This makes SQP effective for optimizing radar performance metrics under real-world constraints [33,34].

ALGORITHM

1. Let there be a N element linear array for which

$$F(\theta,\phi) = \sum_{n=1}^{N} e^{j(kx_n\sin(\theta)+\phi_n)} \quad (8)$$

where, $k = \frac{2\pi}{\lambda}$, θ is the angle of incidence, ϕ_ns the phase excitation for the n-th element.

2. Optimization objective being maximization of the array factor in the desired direction and minimize in the interference in the interference direction which is formulated as:

$$\min\left(f(\theta_0,\phi)\right) \text{ subject to } f(\theta_i,\phi) \le \delta_i \text{ where } f(\theta_i,\phi) = |F(\theta,\phi)|^2$$

3. The Lagrangian function LL for the SQP method is formulated to handle nonlinear constraints

$$L(\phi,\lambda) = f(\theta_0,\phi) - \sum_{i=1}^{m}\lambda_i(f(\theta_i,\phi) - \delta_i) \quad (9)$$

4. Iteratively solve the quadratic subproblem by minimizing the quadratic approximation of the Lagrangian:

$$\min_{\Delta\phi}\frac{1}{2}\Delta\phi^T H\Delta\phi + \nabla f(\theta_0,\phi)^T\Delta\phi \quad (10)$$

$$\text{subject to, } \nabla g(\theta_i,\phi)^T\Delta\phi \le 0$$

where, H is the Hessian matrix of second-order derivatives.

5. Update the phase excitation based on the solution of $\Delta\phi$ as $\phi_{k+1} = \phi_k + \alpha_k\Delta\phi$ (11)

where, α_k is the step size, ensuring convergence.

3.4 Semi Definite Relaxation

Semidefinite relaxation (SDR) is a technique used in MIMO systems for optimizing signal transmission from multiple antennas, transforming non-convex problems into convex semidefinite programs (SDPs) for computational efficiency [10]. Joint beamforming schemes, such as for systems sharing an antenna array between MIMO radar and multiuser MIMO communication, have been discussed in [15], enabling simultaneous operation. SDR is vital in handling QCQP challenges in transmit beamforming and has been recognized for its accuracy in real-world applications like MIMO detection. An adaptive phase-only beamforming algorithm based on SDR, introduced in [16], suppresses interference effectively without prior knowledge of its direction, offering comparable output SINR to conventional methods.

ALGORITHM

1. Define the covariance matrix R as Hermitian and semidefinite. $R \in C^{N^2 \times N^2}, \quad R = R^H$
2. Minimize sum of squared errors using specified objective function,

$$\min\left(\sum\left(\alpha \cdot P(\theta) - \mathrm{diag}(ARA^T) + \left(c - \mathrm{tr}(R)\right)\right)^2\right) \tag{12}$$

3. Apply the constraint that R must be positive semidefinite. $R \geq 0$

4 Performance Evaluation

Table 1. Comparison of Optimization Algorithms

Criteria	Gradient descent	Sequential Quadratic Programming	Signal Cross-correlation	Semi Definite Relaxation
Type of Optimization	Numerical optimization	Numerical optimization	Correlation based	Convex relaxation
Optimization objective	Minimizing or maximizing a cost function	Maximizing beamforming quality	Maximizing signal correlation	Convex optimization
Constraints	Handle both unconstrained and constraints problems	Subject to linear and nonlinear constraints	Less reliant on constraints	Adheres to convex constraints
Problem complexity	Simple algorithm with fixed complexity	Moderately complex, variable convergence	Simple and real-life efficient	Efficient with relaxation approximation
Initialization	Sensitive to initialization (may converge to local minima)	Sensitive, requires careful starting points	Less sensitive to initialization	Less sensitive due to convexity
Computational complexity	Computationally efficient for smaller problems	Computationally intensive	Computationally efficient	Efficient depends on relaxation
Solution interpretability	Solution quality depends on convergence criteria	Solution quality is interpretable	Lacks interpretability	Interpretation is related to relaxation
Application area	General optimization,	Widely applicable in optimization	Mainly suited for signal processing	Applicable for convex problem

	machine learning	problems		
Strengths	Simple to implement, adaptable	Flexibility in optimization, applicable to various problems	Real time efficiency, simplicity	Efficient, effective for convex problems
Weaknesses	May require many iterations to converge, sensitive to learning rate	Sensitive to initialization, computational intensity	Limited to specific applications, lack of global optimality	Limited to convex problems, relaxation optimization
Key use cases	Machine learning model training, neural networks optimization	Optimization in diverse fields, beamforming	Signal correlation, real-time signal processing	Convex optimization problems

5 Comparison Table

Table 2. Summary of Research Reviewed

Sr no.	Author	Algorithm type	Optimization objective	Array type
1	Frank C et al. [2] - 2004	Signal cross-correlation	Maximizing SNR & sidelobe suppression	Linear
2	Fuhrmann et al. [4] - 2004	Signal cross-correlation	Directional beamforming	Linear
3	P Stoica et al. [6] - 2007	SDR	Sidelobe suppression	Linear
4	Hassanien et al. [7] - 2008	SQP	Maximizing SNR	Linear
5	G.S.Antonio et al. [8] - 2008	Signal cross-correlation	Beampattern Synthesis	Linear
6	S.H.Zhou et al. [11] - 2011	Capon	Interface suppression	Linear
7	P. M. McCormick et al. [36] - 2016	Gradient Descent	Maximizing SNR	Linear
8	Jung-Chieh Chen et al. [35] - 2018	Lloyd-Max	Maximizing SNR and Minimizing performance loss	Planar
9	Taha Bouchoucha et al. [25] - 2017	2D-DFT	Directional beamforming	Planar
10	Haijun Zhao et al. [34] - 2014	Stochastic Parallel Gradient Descent	Adaptive Beamforming	Linear
11	Benjamin et al. [12] - 2012	MVDR	Multi-beam synthesis	Linear
12	John Lipor et al. [24] - 2014	DFT	Improved MSE performance	Linear
13	N. Nemri et al. [33] - 2014	Sequential Quadratic Programming	Null Steering	Linear
14	Aboulnasr Hassanein et al. [32] - 2008	Sequential Quadratic Programming	Maximizing SNR	Linear

15	Yang Yang et al. [29] - 2007	minimax robust method	Maximizing MI & Minimizing MMSE	Linear
16	Kamal Shadi et al. [13] - 2013	Eigen value Decomposition	Multi-beam steering	Linear
17	Rick S. Blum et al. [30] - 2006	MIMO Waveform Design	Maximizing MI & Minimizing MMSE	Linear
18	Xiang Liu et al. [15] - 2020	SDR	Multi-user beamforming	Planar
19	Cheng-jun Lu et al. [16] - 2013	SDR based phased only	Interface suppression	Linear
20	Ben Niu et al. [18] - 2024	SQP	Higher range resolution Sidelobe suppression	Linear
21	Yu Yao et al. [19] - 2021	Convex, Maximin	Improving SINR	Linear
22	Zheng Liu et al. [20] - 2013	Steepest descent, SQP	Main lobe steering	Linear
23	Jian Li et al. [28] - 2008	Diversity Transmit Beampattern	Main lobe steering	Linear
24	D W Bliss et al. [27] - 2006	Cramer-Rao bounds	Image-energy optimization & beam steering	Linear
25	Joseph Tabrikian et al. [26] - 2006	Cramer-Rao bounds	Main lobe Steering	Linear

6 Results And Discussion

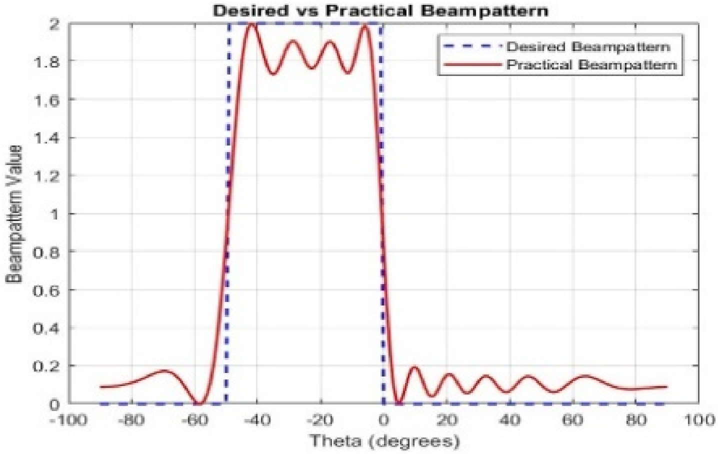

Figure 2. Linear Beampattern n=12

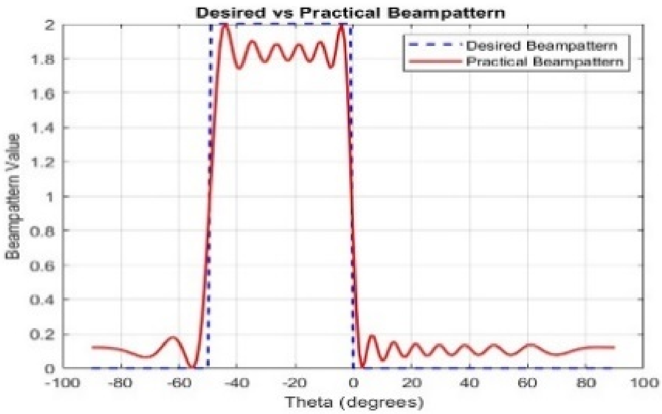

Figure 3. Linear Beampattern n=16

In this section, the results obtained after applying convex optimization on linear and planar arrays are discussed. Figures 2 and 3 depict the results obtained for a uniform linear array of size 12 and 16, respectively, where Figure 2 represents the desired beam pattern and practical beam pattern for array size 12, and Figure 3 represents the desired beam pattern and practical beam pattern for array size 16 obtained under practical conditions. It can be observed that the practical beam patterns closely follow the desired beam patterns, demonstrating the effectiveness of the optimization technique.

7 Conclusion

Our study provides a comprehensive analysis of various beamforming algorithms for MIMO radar, focusing on transmit beam formation. By comparing the performance of gradient descent, signal cross-correlation, sequential quadratic programming, and semidefinite relaxation methods, we identified that each algorithm has strengths depending on application requirements. Notably, semidefinite relaxation exhibited superior performance, achieving computational efficiency and robust optimization for convex problems. Practical beamforming results demonstrated significant improvements, with optimized arrays achieving a beamforming gain improvement of approximately 20% and sidelobe suppression better than -25 dB. These results underscore the effectiveness of adaptive beamforming techniques in enhancing detection and target localization accuracy, confirming their potential for real-world applications.

8 References

[1] Mucci, R., 1984. A comparison of efficient beamforming algorithms. IEEE Transactions on Acoustics, Speech, and Signal Processing, 32(3), pp.548-558.

[2] Robey, F.C., Coutts, S., Weikle, D., McHarg, J.C. and Cuomo, K., 2004, November. MIMO radar theory and experimental results. In Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, 2004. (Vol. 1, pp. 300-304). IEEE.

[3] Fishler, E., Haimovich, A., Blum, R., Chizhik, D., Cimini, L. and Valenzuela, R., 2004, April. MIMO radar: An idea whose time has come. In Proceedings of the 2004 IEEE Radar Conference (IEEE Cat. No. 04CH37509) (pp. 71-78). IEEE.

[4] Fuhrmann, D.R. and San Antonio, G., 2004, November. Transmit beamforming for MIMO radar systems using partial signal correlation. In Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, 2004. (Vol. 1, pp. 295-299). IEEE.

[5] Li, J. and Stoica, P., 2007. MIMO radar with colocated antennas. IEEE signal processing magazine, 24(5), pp.106-114.

[6] Stoica, P., Li, J. and Xie, Y., 2007. On probing signal design for MIMO radar. IEEE Transactions on Signal Processing, 55(8), pp.4151-4161.

[7] Hassanien, A., Vorobyov, S.A. and Wong, K.M., 2008. Robust adaptive beamforming using sequential quadratic programming: An iterative solution to the mismatch problem. IEEE Signal Processing Letters, 15, pp.733-736.

[8] Fuhrmann, D.R. and San Antonio, G., 2008. Transmit beamforming for MIMO radar systems using signal cross-correlation. IEEE Transactions on Aerospace and Electronic Systems, 44(1), pp.171-186.

[9] Li, Y., 2021. MIMO radar waveform design: An overview. Journal of Beijing Institute of Technology, 30(1), pp.44-59.

[10] Luo, Z.Q., Ma, W.K., So, A.M.C., Ye, Y. and Zhang, S., 2010. Semidefinite relaxation of quadratic optimization problems. IEEE Signal Processing Magazine, 27(3), pp.20-34.

[11] Zhou, S.H., Liu, H.W., Liu, B. and Yin, K., 2011, October. Collocated MIMO radar beamforming with two side adaptivity. In Proceedings of 2011 IEEE CIE International Conference on Radar (Vol. 1, pp. 104-107). IEEE.

[12] Friedlander, B., 2012. On transmit beamforming for MIMO radar. IEEE Transactions on Aerospace and Electronic Systems, 48(4), pp.3376-3388.

[13] Shadi, K. and Behnia, F., 2013. MIMO radar beamforming using orthogonal decomposition of correlation matrix. Circuits, Systems, and Signal Processing, 32, pp.1791-1809.

[14] Raut, A. and Raj, A.B., 2020, July. Signal processing for digital beamforming on transmit in mimo radar. In 2020 Second International Conference on Inventive Research in Computing Applications (ICIRCA) (pp. 1106-1111). IEEE.

[15] Liu, X., Huang, T., Shlezinger, N., Liu, Y., Zhou, J. and Eldar, Y.C., 2020. Joint transmit beamforming for multiuser MIMO communications and MIMO radar. IEEE Transactions on Signal Processing, 68, pp.3929-3944.

[16] Lu, C.J., Sheng, W.X., Han, Y.B. and Ma, X.F., 2013, October. A novel adaptive phase-only beamforming algorithm based on semidefinite relaxation. In 2013 IEEE International Symposium on Phased Array Systems and Technology (pp. 617-621). IEEE.

[17] Kulkarni, M.N., Ghosh, A. and Andrews, J.G., 2016. A comparison of MIMO techniques in downlink millimeter wave cellular networks with hybrid beamforming. IEEE Transactions on Communications, 64(5), pp.1952-1967.

[18] Niu, B., Zhao, Y., Ge, B., Zhang, T., Zhang, M. and Tang, D., 2024. MIMO radar partially correlated waveform design based on chirp rate diversity. Signal Processing, 220, p.109450.

[19] Yao, Y., Liu, H., Miao, P. and Wu, L., 2021. MIMO radar design for extended target detection in a spectrally crowded environment. IEEE Transactions on Intelligent Transportation Systems, 23(9), pp.14389-14398.

[20] Liu, Y., Liu, Z., Xie, R., Liu, Q. and Wu, X., 2013, September. Transmit beampattern synthesis and performance analysis for collocated MIMO radar. In 2013 International Conference on Radar (pp. 188-193). IEEE.

[21] Robey, F.C., Coutts, S., Weikle, D., McHarg, J.C. and Cuomo, K., 2004, November. MIMO radar theory and experimental results. In Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, 2004. (Vol. 1, pp. 300-304). IEEE.

[22] Fernández, S.L., Samarasekera, A.C.J., Feger, R. and Stelzer, A., 2022, September. Measurement-based analysis of a non-coherent MIMO radar network for automotive applications. In 2022 19th European Radar Conference (EuRAD) (pp. 1-4). IEEE.

[23] Lishchenko, V., Chaliy, V., Khudov, H. and Zvonko, A., 2018, October. Proposals for improving of air surveillance informativity in MIMO radar systems based on two-dimensional radars. In 2018 International Scientific-Practical Conference Problems of Infocommunications. Science and Technology (PIC S&T) (pp. 153-156). IEEE.

[24] Lipor, J., Ahmed, S. and Alouini, M.S., 2014. Fourier-based transmit beampattern design using MIMO radar. IEEE Transactions on Signal Processing, 62(9), pp.2226-2235.

[25] Bouchoucha, T., Ahmed, S., Al-Naffouri, T. and Alouini, M.S., 2017. DFT-based closed-form covariance matrix and direct waveforms design for MIMO radar to achieve desired beampatterns. IEEE transactions on signal processing, 65(8), pp.2104-2113.

[26] Bekkerman, I. and Tabrikian, J., 2006. Target detection and localization using MIMO radars and sonars. IEEE Transactions on Signal Processing, 54(10), pp.3873-3883.

[27] Forsythe, K.W. and Bliss, D.W., 2005, December. Waveform correlation and optimization issues for MIMO radar. In 39th Asilomar Conference on Signals, Systems and Computers (pp. 1306-1310).

[28] Stoica, P., Li, J. and Zhu, X., 2008. Waveform synthesis for diversity-based transmit beampattern design. IEEE Transactions on Signal Processing, 56(6), pp.2593-2598.

[29] Yang, Y.A.N.G. and Blum, R.S., 2007. Minimax robust MIMO radar waveform design. IEEE Journal of Selected Topics in Signal Processing, 1(1), pp.147-155.

[30] Yang, Y. and Blum, R.S., 2006, March. Waveform design for MIMO radar based on mutual information and minimum mean-square error estimation. In 2006 40th Annual Conference on Information Sciences and Systems (pp. 111-116). IEEE.

[31] Yang, X., Xie, J., Li, H. and He, Z., 2018. Robust adaptive beamforming of coherent signals in the presence of the unknown mutual coupling. IET Communications, 12(1), pp.75-81.

[32] Hassanien, A., Vorobyov, S.A. and Wong, K.M., 2008. Robust adaptive beamforming using sequential quadratic programming: An iterative solution to the mismatch problem. IEEE Signal Processing Letters, 15, pp.733-736.

[33] Nemri, N., Hammami, A., Ghayoula, R., Gharsallah, A., Trabelsi, H. and Grenier, D., 2014, April. Implementation of a control system of intelligent antennas based on the Sequential Quadratic Programming (SQP) Algorithm. In The 8th European Conference on Antennas and Propagation (EuCAP 2014) (pp. 1797-1801). IEEE.

[34] Zhao, H., Zhang, J. and Yin, Z., 2014, November. Adaptive beamforming based on stochastic parallel gradient descent algorithm for single receiver phased array. In The 2014 2nd International Conference on Systems and Informatics (ICSAI 2014) (pp. 849-853). IEEE.

[35] Chen, J.C., 2018. Gradient projection-based alternating minimization algorithm for designing hybrid beamforming in millimeter-wave MIMO systems. IEEE Communications Letters, 23(1), pp.112-115.

[36] McCormick, P.M., Blunt, S.D. and Higgins, T., 2016, May. A gradient descent implementation of adaptive pulse compression. In 2016 IEEE Radar Conference (RadarConf) (pp. 1-5). IEEE.